Problem of the week

Atomic Physics (HL only)

(a) An alpha particle of initial speed *u* is directed head on at a nucleus of proton number *Z*. The distance of closest approach is *d*.

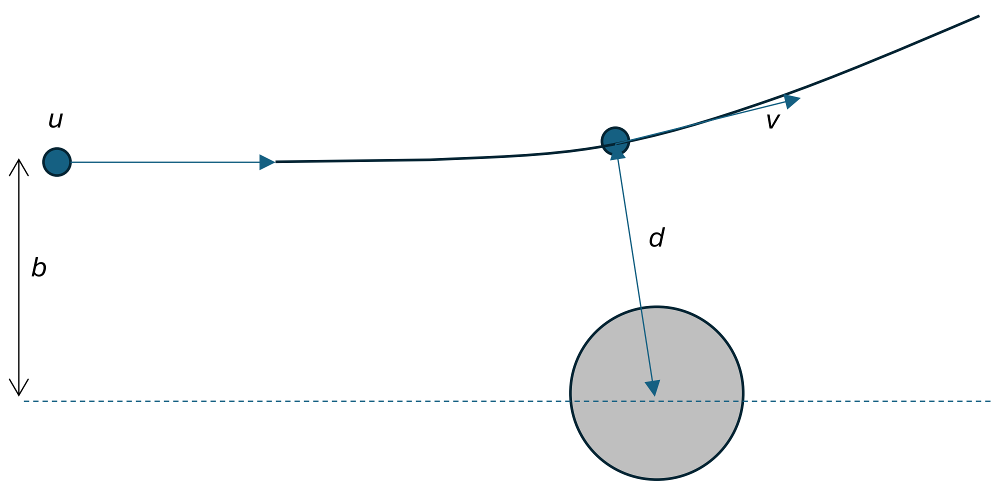
(i) Show that $u^2 = \frac{4kZe^2}{m_\alpha d}$.

(ii) The following data are given:

$u = 1.36\times10^{7}\ \text{m s}^{-1}$
$Z = 32$
$m_\alpha = 6.64\times 10^{-27}\ \text{kg}$

Determine *d*.

(b) In another experiment an alpha particle of initial speed *u* is directed at a nucleus and follows the path shown. The distance of closest approach is *d*. At this distance, the speed of the alpha particle is *v*. The impact parameter (the vertical distance between the center of the nucleus and the initial velocity of the alpha particle) is equal to *b*.



Explain why $ub = vd$.

(c) A second alpha particle approaches the same nucleus with the same initial speed u as in (b). The impact parameter is $\frac{b}{2}$.

(i) Sketch the path of this alpha particle on the diagram in (b).

(ii) Suggest whether the distance of closest approach will be less than, equal to or greater than $\frac{d}{2}$.

(d) For the Rutherford-Marsden-Geiger experiment,

(i) suggest what is meant by *deviations* from Rutherford scattering,

(ii) state the reason why deviations from Rutherford scattering occur.

(e) Explain why the density of a nucleus of nucleon number A is independent of A.

(f) Bohr introduced the condition $mv_n r_n = n\frac{h}{2\pi}$ for the electron in the hydrogen atom. Explain the reasoning for this condition.

(g) In the Bohr model of hydrogen, the electron radius is given by $r_n = a_0 n^2$ where a_0 is a constant. Show that for the nth state

(i) the electron speed is $v_n = \frac{h}{2\pi m a_0}\frac{1}{n}$,

(ii) the electron period of revolution is $T_n = \frac{4\pi^2 m a_0^2}{h} n^3$,

(iii) hence or otherwise deduce Kepler's third law, i.e. that $T_n^2 \sim r_n^3$.

Answers

(a)

(i) Conservation of energy gives $\frac{1}{2}m_{\alpha}u^2 = 0 + \frac{2kZe^2}{d}$ hence the result.

(ii)

$$d = \frac{4kZe^2}{m_{\alpha}u^2}$$

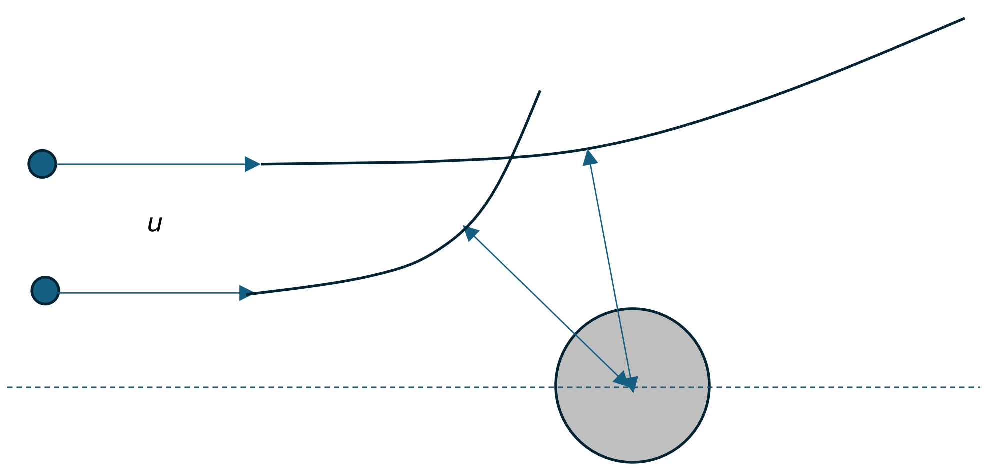
$$d = \frac{4 \times 8.99 \times 10^9 \times 32 \times \left(1.6 \times 10^{-19}\right)^2}{6.64 \times 10^{-27} \times \left(1.36 \times 10^7\right)^2}$$

$$d = 2.40 \times 10^{-14}\ \mathrm{m}$$

(b) Conservation of angular momentum demands $m_{\alpha}ub = m_{\alpha}vd$. Angular momentum is conserved because the only force on the alpha particle is the electric force along the line joining the centers of the alpha and the nucleus, so this force has no torque.

(c)

(i)



(ii) From $ub = vd$, the left-hand side is halved. If the speed *v* at the point of closest approach stayed the same the distance of closest approach would become $\frac{d}{2}$. But this speed decreases so *d* will be greater than $\frac{d}{2}$ (and less than *d*).

(d)

(i) The fraction of alpha particles scattered at a specific angle is different from what the Rutherford model predicts.

(ii) The Rutherford model assumes that only the electric force acts on the alpha particles. As the energy of the alpha particles increases the distance of closest approach decreases and the alphas come within the range of the strong nuclear force.

(e) The density is given by $\rho = \frac{m}{V}$. A nucleus with nucleon number A has a mass that is approximately Au. Because the radius is given by $R = R_0 A^{\frac{1}{3}}$ the volume is $V = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}R_0^3 A$, hence the density is $\rho = \frac{Au}{\frac{4\pi}{3}R_0^3 A} = \frac{u}{\frac{4\pi}{3}R_0^3}$, independent of A.

(f) Bohr needed to correct for the fact that the electron in a circular orbit accelerates and so radiates energy away in the form of EM waves. This would make the electron plunge into the nucleus. Bohr postulated (without justification) that in orbits where the condition $mv_n r_n = n\frac{h}{2\pi}$ is satisfied, the electron would not radiate and so matter would be stable.

(g)

(i) Substitute $r_n = a_0 n^2$ in $mv_n r_n = n\frac{h}{2\pi}$ to get $mv_n a_0 n^2 = n\frac{h}{2\pi}$ and so $v_n = \frac{h}{2\pi m a_0}\frac{1}{n}$.

(ii) $T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi a_0 n^2}{\frac{h}{2\pi m a_0}\frac{1}{n}} = \frac{4\pi^2 m a_0^2}{h}n^3$.

(iii) $T_n^2 \sim n^6$ and $r_n^3 \sim n^6$, hence $T_n^2 \sim r_n^3$.